

No Cosmic Rays from Curvature Oscillations during Structure Formation with $F(R)$ -gravity

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Abstract

The Starobinsky model of modified gravity suggested to explain dark energy may be also considered in the astrophysical context. Recently it has been pointed out that in contracting regions curvature oscillations around the GR value may lead to the production of high energy particles which contribute to the cosmic ray flux. We revisit these calculations in the Einstein frame and show that the continuous approximation for the matter density used in the original calculations is not valid. We go beyond this approximation and find the rate of particle production to be negligible.

1 Introduction

Theories of modified gravity have been suggested as models capable of explaining the accelerated expansion of the present Universe. The model of $F(R)$ -gravity proposed by Starobinsky [1] successfully describes an evolution of the late-time Universe imitating the cosmological constant. The corresponding action for gravity reads

$$S = -\frac{M_{\text{P}}^2}{2} \int d^4x \sqrt{-g} (R + F(R)), \quad (1)$$

where the reduced Planck mass M_{P} is expressed via the Newtonian gravitational constant G_N as follows $M_{\text{P}} = \sqrt{8\pi/G_N} = 2.4 \times 10^{18} \text{ GeV}$, and

$$F(R) = \lambda R_0 \left(\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right) - \frac{R^2}{6m^2}. \quad (2)$$

Here m is the maximal mass of additional scalar degree of freedom, *scalaron*, that appears in $F(R)$ gravity, and parameter R_0 fixes the scale of energy density associated with the cosmological constant ρ_Λ at present, when the Hubble parameter equals $H_0 = 1.45 \times 10^{-42} \text{ GeV}$,¹

$$R_0 = -\frac{2}{\lambda} \frac{\rho_\Lambda}{M_P^2} = -\frac{6}{\lambda} \frac{\Omega_\Lambda H_0^2}{\lambda}, \quad (3)$$

with $\Omega_\Lambda \equiv \rho_\Lambda/\rho_c$ and critical energy density ρ_c determined by the Friedman equation,

$$3 M_P^2 H_0^2 = \rho_c. \quad (4)$$

The main question is how to distinguish $F(R)$ -gravity from the cosmological constant and other Dark Energy models? One way is to probe the Dark Energy equation of state: $F(R)$ -gravity may predict Dark Energy with time-dependent and even phantom pressure-to-density relation (see e.g. [2, 3, 4] for reviews). Another way is to look for possible observable effects in astrophysics.

$F(R)$ -gravity is equivalent to usual gravity and an additional scalar field with a non-trivial potential and coupling to matter fields. In papers [5, 6] it was pointed out that, in space regions with rising matter density, growing curvature oscillations decay into high energy particles, thus contributing to the Cosmic Ray spectrum. These oscillations are oscillations of classical scalaron field. In this paper we address a question: where do these scalaron oscillations come from? Usually, a scalar field oscillates around minimum if for some reason it is shifted or pushed from the vacuum. In papers [5, 6] the initial amplitude of the oscillations is arbitrary while actually there is no arbitrariness because the history of the expanding Universe exactly defines the initial conditions for the scalaron field. Namely, scalaron being massive in the early Universe, decays to the Standard model particles providing the absence of scalaron excitations at the present time. However, scalarons may be produced through the quantum processes [7]. We show below that numerical estimates in [5, 6], performed for the adopted there initial conditions, correspond to very unrealistic situation of huge number density (and noticeable energy density) of the present scalaron configuration.

Actually, as it was noticed in [5], the oscillations arise even if the field is adiabatically settled in the vacuum because the scalaron minimum itself moves with rising density. However, this oscillation source leads to parametrically smaller amplitudes than those considered in [5, 6]. For matter densities close to the present dark energy (and for parameters in a particular region) the oscillations might contribute a noticeable amount to the energy density

¹This simple relation is valid for $\lambda \gg 1$.

of Universe (this situation, if realistic, needs a special study). However, this energy density does not release as high energy particles. We revisit calculations of [5, 6] and obtain that the used approximation of homogeneous matter and scalaron field does not work in the region where significant particle production was predicted. We go beyond this approximations and estimate the realistic flux of produced cosmic rays to be negligible.

2 Scalaron density in contracting objects

Classical oscillations of scalaron field ϕ may be described in terms of scalaron condensate and the number density of scalarons n_ϕ is defined through the amplitude of oscillations as [8]

$$n_\phi = \omega \langle \phi^2 \rangle. \quad (5)$$

Here ϕ is a canonically normalized scalaron field and ω is the energy of each particle in the condensate, i.e. the *scalaron effective mass*. It depends on the surrounding matter density ρ_m as [7]²

$$\omega = \frac{H_0}{\sqrt{2n\lambda(2n+1)}} \left(\frac{\rho_m}{\rho_c} \right)^{n+1} \left(\frac{\lambda}{2\Omega_\Lambda} \right)^{n+1/2}. \quad (6)$$

While an astrophysical object (i.e. halo) contracts, ρ_m grows. In [5, 6] matter density ρ_m changes linearly with time as $\rho_m = \rho_{m0}(1 + t/t_J)$ for $t < t_J$ with t_J being the Jeans time of contraction.

The scalaron field is related to the scalar curvature and may be defined through the derivative $F'(R) \equiv dF/dR$ as

$$\phi = \frac{\sqrt{6}}{2} M_P \log(1 + F'(R)) \approx \frac{\sqrt{6}}{2} M_P F'(R). \quad (7)$$

The equations of motion for action (1) provide with the equation of motion for scalaron. It is convenient to write this equation in terms of dimensionless variables which first has been done in [9]. Later calculations in [5, 6] are performed in terms of variable ξ connected with scalaron field ϕ as follows,

$$\xi = -\frac{1}{2\lambda n} \left(\frac{\rho_{m0}}{M_P^2 R_0} \right)^{2n+1} F'(R) = -\frac{1}{2\lambda n} \left(\frac{\lambda \rho_{m0}}{2\Omega_\Lambda \rho_c} \right)^{2n+1} F'(R). \quad (8)$$

The equation of motion for ξ , that defines the time dependence of scalaron field, reads [5]

$$\xi'' + \frac{\rho_m}{\rho_{m0}} - y = 0, \quad (9)$$

²This dependence is valid until $\omega \ll m$. In the opposite limit $\omega \approx m$.

where $y = y(\xi)$ is obtained from equation ³

$$\frac{1}{y^{2n+1}} - gy = \xi, \quad g \equiv \frac{H_0^2}{2n\lambda m^2} \left(\frac{\rho_{m0}}{\rho_c} \right)^{2n+2} \left(\frac{\lambda}{2\Omega_\Lambda} \right)^{2n+1}. \quad (10)$$

Primes in (9) correspond to the derivatives with respect to dimensionless time $\tau \equiv m\sqrt{g}t$, and numerically $g \ll 1$.

Scaloron field ϕ may be expressed in terms of function ξ by using (7) and (8)

$$\phi = \sqrt{6}\lambda n M_P \left(\frac{2\Omega_\Lambda \rho_c}{\lambda \rho_{m0}} \right)^{2n+1} \xi. \quad (11)$$

As it is shown in the paper [5] when initial conditions expected in General Relativity are imposed, $y = 1$ (which implies for the scalaron to be in the minimum of the potential), the amplitude of ξ -oscillations becomes

$$\delta\xi = (\kappa - y'_0)(2n+1)^{3/2}, \quad \kappa = \sqrt{6\lambda n} \left(\frac{\rho_{m0}}{\rho_c} \right)^{-n-1/2} \left(\frac{\lambda}{2\Omega_\Lambda} \right)^{-n-1/2}. \quad (12)$$

Here (in the limit of small g) y'_0 is proportional to the derivative of curvature R and is considered as a free parameter in [5]. Since the case of $y'_0 = \kappa$ is recognized in [5] as fine tuning, the initial amplitude is found to be of order $\delta\xi = \kappa(2n+1)^{3/2}$. Putting all things together we obtain the initial energy density of the scalaron condensate:

$$\rho_\phi(t=0) = \langle \phi^2 \rangle \omega^2(t=0) = 9\lambda^2 n^2 (2n+1)^2 M_P^2 H_0^2 \left(\frac{2\Omega_\Lambda}{\lambda} \right)^{4n+2} \left(\frac{\rho_c}{\rho_{m0}} \right)^{4n+1}. \quad (13)$$

Accounting for the Friedman equation (4) we estimate for the set of parameters considered in [5] ($n = 2$, $\lambda = 1$, $\kappa = 0.04$) the initial scalaron energy density

$$\rho_\phi(t=0) = 1.5 \times 10^{-4} \rho_c, \quad (14)$$

that actually exceeds the radiation (CMB) energy density at present. However one cannot expect such a large contribution of scalarons because less than one particle inside the horizon may be created in the present (or recent) Universe, see [7]. Scalarons created in the very early Universe were very heavy (with mass m) and hence decayed to the SM particles. So the initial conditions in [5] seem to be irrelevant.

³Here we use the value of R_0 defined by (3); in [5] it is taken approximately as $R_0 = -1/t_U^2$ with t_U being the Universe age. This results in different dependence of g on λ and Ω_Λ , as compared to [5], which is not important, however.

3 Relevant initial conditions for scalaron field

When density changes in a contracting object the form of scalaron potential changes: its minimum goes closer to $\phi = 0$ and its mass rises. The initial condition $y'_0 = 0$ in [6] implies that scalaron at $t = 0$ is put into the moving minimum with zero 'velocity'. But actually we should expect that at $t < 0$ (i.e. before the contraction starts) there were no excitations and scalaron was in the vacuum state. Also we should propose that in the real situation the contraction starts in a smooth way providing the adiabatic evolution near $t = 0$ [7] and oscillations should be excited with the minimal possible amplitude. In what follows it is convenient to introduce dimensionless time $\tau = t/(\kappa t_J)$, where t_J is the Jeans time, and new variables

$$\bar{\xi} = \xi - \xi_{min}, \quad (15)$$

where

$$\xi_{min} = (1 + \kappa\tau)^{-(2n+1)}. \quad (16)$$

So the *adiabatic* solution of the scalaron equation of motion derived in [5],

$$\bar{\xi}'' + \Omega^2 \bar{\xi} = -\xi_{min}'' , \quad (17)$$

with such (zero) initial conditions that at $t = 0$ one has $\bar{\xi} = 0$, $\bar{\xi}' = 0$ looks to be the closest to the realistic physical situation. Here we use notations of paper [5],

$$\Omega \equiv \frac{(1 + \kappa\tau)^{n+1}}{\sqrt{2n+1}}, \quad (18)$$

and primes correspond to derivatives with respect to τ .⁴

Note that the source in the r.h.s. of (17) even with zero initial conditions leads to oscillations of $\bar{\xi}$. The adiabatic solution of (17) may be obtained by the standard technique in the form

$$\bar{\xi} = -\xi_1 \int \xi_2 \xi_{min}'' d\tau + \xi_2 \int \xi_1 \xi_{min}'' d\tau, \quad (19)$$

where

$$\xi_1 = \frac{1}{\sqrt{\Omega}} \sin \int \Omega d\tau, \quad \xi_2 = \frac{1}{\sqrt{\Omega}} \cos \int \Omega d\tau. \quad (20)$$

The solution (19) may be rewritten in the form (s.f. eq. (12) of Ref. [5])

$$\bar{\xi} = \alpha(\tau) \sin \left(\int \Omega d\tau + \delta(\tau) \right). \quad (21)$$

⁴We treat maximal scalaron mass m as being very large compared to the scale of cosmological constant, so the parameter g used in [5] may be set to zero in this case.

After some calculations one finds the amplitude $\alpha(\tau)$ of generated oscillations in the limit of small $\kappa \ll 1$,

$$\alpha(\tau) \simeq C_n \kappa^2 (1 + \kappa \tau)^{-\frac{n+1}{2}}, \quad C_n = 2(n+1)(2n+1)^2. \quad (22)$$

This result corresponds to the initial amplitude of oscillations equal to $\alpha_0 = C_n \kappa^2$, not of order κ as it is proposed in [5]. So we obtain parametrically smaller amplitude for matter densities much larger than the critical one (astrophysical processes), $\rho_{m0} \gg \rho_c$, while for matter densities close to the critical one (cosmological processes), $\rho_{m0} \sim \rho_c$, the amplitude may be large, see eq. (11), providing (potentially) a significant contribution to the energy density of the Universe (13) for some choice of parameters: e.g., $n = 2$ and $\lambda \approx 1$. In this region of model parameter space the detailed analysis of theoretical consistency and phenomenological and cosmological viability of $F(R)$ -model is needed.

As noted in [5, 6] scalaron oscillations may produce massless particles. But when $\xi > 0$ (regular region, see eqs. (10), (9)) the process cannot be efficient because of very small effective scalaron mass. We show that using the approximation of decaying scalaron condensate which works perfectly well in the regular region. The scalaron density for the amplitude (22) is of order

$$\rho \sim M_P^2 H_0^2 \left(\frac{\rho_{m0}}{\rho_c} \right)^{-6n-2}. \quad (23)$$

We use the scalaron decay width to (massless) gauge bosons from [13], $\Gamma \sim \omega^3/M_P^2$, and estimate the energy density of particles created by scalaron oscillations during the Universe lifetime as

$$\rho_p \sim \frac{\rho \Gamma}{H_0} = H_0^4 \left(\frac{\rho_{m0}}{\rho_c} \right)^{-3n+1}, \quad (24)$$

which in any case means less than one particle in a horizon-size region. The reason here is clear, since $\omega \sim H_0$, the scalarons are stable at the cosmological time-scale, $\Gamma \sim H_0(H_0/M_P)^2 \ll H_0$. A similar result has been obtained in [6].

Only if ξ becomes negative (in a spike region as it is called in [6]) its effective mass may increase because the scalaron potential at negative ϕ may be very steep. One observes from (12), (15), (16), (22), that the spike region can be reached on the Jeans time scale only for matter densities ρ_m close to ρ_c , and hence is relevant only for the recent cosmic structure formation.

In the next Section 4 we estimate the flux of particles produced by scalaron oscillations in the spike region ($\xi < 0$).

4 Particle production in the spike region

Hereafter we consider the case of matter densities ρ_m close to ρ_c which may help ξ to reach negative values. In this region of parameters the time of contraction is very large, of order of the Universe age and the contraction has been started not long ago. So we are interested in the evolution only until t_J , and for that time one has $\Omega\kappa t \lesssim 1$, see Eq. (18), and ξ reaches negative values only once or twice each time producing a spike in the solution for curvature [5, 6]. Below we estimate the particle production during one spike.

Homogeneous oscillations of classical scalaron field may produce non-conformal particles and production of scalars minimally coupled to gravity is the most efficient [10]. Scalaron is coupled to scalar field h through the kinetic mixing in lagrangian [10]

$$L_{int} = \frac{h}{\sqrt{6}M_P} \partial_\mu \phi \partial^\mu h. \quad (25)$$

This coupling modifies the equation of motion for h :

$$\partial_\mu \partial^\mu h + \left(\frac{\ddot{\phi}}{\sqrt{6}M_P} + m_h^2 \right) h = 0, \quad (26)$$

where m_h is the scalar mass. Scalaron field here plays a role of the external force producing particles h . The number of produced particles may be estimated by making use of the Bogoloubov transformations. From eq. (26) one obtains for the Fourier mode h_k of 3-momentum k ,

$$\ddot{h}_k + (k^2 + m_{eff}^2(t) + m_h^2) h_k = 0, \quad (27)$$

where $m_{eff}^2(t) = \ddot{\phi}/(\sqrt{6}M_P)$.

Note that the spikes correspond to the negative values of ϕ and in that region the scalaron mass is maximal, so one has $\ddot{\phi} = -m^2\phi$ during the spike. The maximum value of $|\phi_m|$ may be extracted from the maximum $|\xi_m|$ estimated in [5]:

$$\frac{|\phi_m|}{\sqrt{6}M_P} = N \frac{H_0}{m} \left(\frac{\rho_{m0}}{\rho_c} \right)^{-n} z_1^{-n}, \quad N = \sqrt{\frac{\lambda}{2}} \left(\frac{2\Omega_\Lambda}{\lambda} \right)^{n+1/2}. \quad (28)$$

Here $z_1 \equiv 1 + t_1/t_J$ where t_1 corresponds to the moment when ξ crosses zero for the first time: $\alpha(\tau_1) = \xi_{min}(\tau_1)$. Note that for the case we consider the variable z is in the interval $1 < z_1 < 2$, so we put it to be $z_1 = 1$ hereafter having in mind an upper bound. Then the maximum possible value of $m_{eff}^2(t)$ is

$$M^2 = NmH_0 \left(\frac{\rho_{m0}}{\rho_c} \right)^{-n}. \quad (29)$$

If we approximate the spike by the gaussian form with height of M and width of $1/m$ we obtain formally that the spectrum has a cut off at high momenta of order m . The calculation shows that high energy particles with number density of order $n_p \sim M^4/m$ are produced by spike, in agreement with the statement of Ref. [5].

Since production of such energetic particles in a slow and smooth process of contraction looks very surprising from any point of view it is needed to check the validity of all approximations in use. The approximation that is certainly doubted is the homogeneous energy-momentum tensor for the background matter. If one considers a set of discrete particles instead of continuous medium one obtain that they produce the scalaron field like point sources. Every particle produces the field $\xi \propto e^{-\omega r}/r$ like the usual massive scalar with mass ω . If the average distance d between particles is large, $d \gg 1/\omega$, then the scalaron field is strongly inhomogeneous and approximation used for the energy-momentum tensor is not valid.

In this case we expect that the energetic particle production may still take place only in the small vicinities of the point sources, i.e. in very small spherical regions of radius $1/m$. Therefore, the total number of produced particles is suppressed by a very small number n/m^3 where n is the matter number density. For the energy density we get the estimate

$$\rho \sim \frac{M^4 n}{m^3} = N^2 \rho_{m0} \frac{H_0^2}{m_0 m} \left(\frac{\rho_{m0}}{\rho_c} \right)^{-2n}, \quad (30)$$

where m_0 is the mass of particle populating the contracting object (region). For the interesting astrophysical objects and any regions dominated by baryons one puts $m_0 \sim 1 \text{ GeV}$. Then for the close to critical density and the scalaron mass $m \sim 10^{13} \text{ GeV}$ we obtain $\rho \sim 10^{-92} \text{ GeV}/(\text{cm}^2 \text{ s})$ which is too small number to talk about.

Note in passing, though we discussed the particle production from one spike, the consideration of a sequence of several spikes does not significantly change the result. One more comment concerns the contraction of dark matter dominated region. To the case of particle-like dark matter (e.g. weakly interaction massive particles) the estimate (30) is fully applicable with m_0 referring to the dark matter particle mass. The case of homogeneous scalar field playing the role of dark matter is inconsistent [14] with stability of the studied in [5, 6] $F(R)$ -gravity (2) at small R and is beyond the scope of this paper.

To summarize, we have shown that contracting objects made of baryons or dark matter particles in $F(R)$ -gravity do not contribute to the cosmic ray spectrum via the decay of the scalaron oscillations.

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